Chapter 16
Electrical Energy and Capacitance

Quick Quizzes

1. (b). The field exerts a force on the electron, causing it to accelerate in the direction opposite to that of the field. In this process, electrical potential energy is converted into kinetic energy of the electron. Note that the electron moves to a region of higher potential, but because the electron has negative charge this corresponds to a decrease in the potential energy of the electron.

2. (b), (d). Charged particles always tend to move toward positions of lower potential energy. The electrical potential energy of a charged particle is $PE = qV$ and, for positively-charged particles, this increases as V increases. For a negatively-charged particle, the potential energy decreases as V increases. Thus, a positively-charged particle located at $x = A$ would move toward the left. A negatively-charged particle would oscillate around $x = B$ which is a position of minimum potential energy for negative charges.

3. (d). If the potential is zero at a point located a finite distance from charges, negative charges must be present in the region to make negative contributions to the potential and cancel positive contributions made by positive charges in the region.

4. (c). Both the electric potential and the magnitude of the electric field decrease as the distance from the charged particle increases. However, the electric flux through the balloon does not change because it is proportional to the total charge enclosed by the balloon, which does not change as the balloon increases in size.

5. (a). From the conservation of energy, the final kinetic energy of either particle will be given by

$$KE_f = KE_i + \left(PE_i - PE_f\right) = 0 + qV_i - qV_f = -q\left(V_f - V_i\right) = -q(\Delta V)$$

For the electron, $q = -e$ and $\Delta V = +1\text{ V}$ giving $KE_f = -(-e)(+1\text{ V}) = +1\text{ eV}$.

For the proton, $q = +e$ and $\Delta V = -1\text{ V}$, so $KE_f = -(e)(-1\text{ V}) = +1\text{ eV}$, the same as that of the electron.

6. (c). The battery moves negative charge from one plate and puts it on the other. The first plate is left with excess positive charge whose magnitude equals that of the negative charge moved to the other plate.

7. **(a)** C decreases. **(b)** Q stays the same. **(c)** E stays the same. **(d)** ΔV increases. **(e)** The energy stored increases.

Because the capacitor is removed from the battery, charges on the plates have nowhere to go. Thus, the charge on the capacitor plates remains the same as the plates are pulled apart. Because $E = \frac{\sigma}{\epsilon_0} = \frac{Q/A}{\epsilon_0}$, the electric field is constant as the plates are separated. Because $\Delta V = Ed$ and E does not change, ΔV increases as d increases. Because the same charge is stored at a higher potential difference, the capacitance has decreased. Because $\text{Energy stored} = Q^2/2C$ and Q stays the same while C decreases, the energy stored increases. The extra energy must have been transferred from somewhere, so work was done. This is consistent with the fact that the plates attract one another, and work must be done to pull them apart.

8. **(a)** C increases. **(b)** Q increases. **(c)** E stays the same. **(d)** ΔV remains the same. **(e)** The energy stored increases.

The presence of a dielectric between the plates increases the capacitance by a factor equal to the dielectric constant. Since the battery holds the potential difference constant while the capacitance increases, the charge stored $(Q = C\Delta V)$ will increase. Because the potential difference and the distance between the plates are both constant, the electric field $(E = \Delta V/d)$ will stay the same. The battery maintains a constant potential difference. With ΔV constant while capacitance increases, the stored energy $\left(\text{Energy stored} = \frac{1}{2}C(\Delta V)^2\right)$ will increase.

9. (a). Increased random motions associated with an increase in temperature make it more difficult to maintain a high degree of polarization of the dielectric material. This has the effect of decreasing the dielectric constant of the material, and in turn, decreasing the capacitance of the capacitor.

Answers to Even Numbered Conceptual Questions

2. Changing the area will change the capacitance and maximum charge but not the maximum voltage. The question does not allow you to increase the plate separation. You can increase the maximum operating voltage by inserting a material with higher dielectric strength between the plates.

4. Electric potential V is a measure of the potential energy per unit charge. Electrical potential energy, $PE = QV$, gives the energy of the total charge Q.

6. A sharp point on a charged conductor would produce a large electric field in the region near the point. An electric discharge could most easily take place at the point.

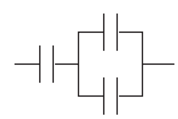
8. There are eight different combinations that use all three capacitors in the circuit. These combinations and their equivalent capacitances are:

All three capacitors in series - $C_{eq} = \left(\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}\right)^{-1}$

All three capacitors in parallel - $C_{eq} = C_1 + C_2 + C_3$

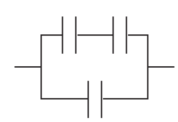
One capacitor in series with a parallel combination of the other two:

$$C_{eq} = \left(\frac{1}{C_1 + C_2} + \frac{1}{C_3}\right)^{-1},\ C_{eq} = \left(\frac{1}{C_3 + C_1} + \frac{1}{C_2}\right)^{-1},\ C_{eq} = \left(\frac{1}{C_2 + C_3} + \frac{1}{C_1}\right)^{-1}$$



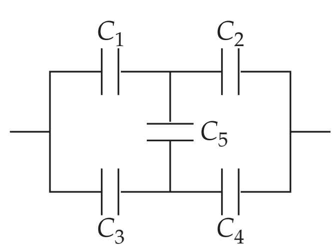
One capacitor in parallel with a series combination of the other two:

$$C_{eq} = \left(\frac{C_1C_2}{C_1 + C_2}\right) + C_3,\ C_{eq} = \left(\frac{C_3C_1}{C_3 + C_1}\right) + C_2,\ C_{eq} = \left(\frac{C_2C_3}{C_2 + C_3}\right) + C_1$$



10. Nothing happens to the charge if the wires are disconnected. If the wires are connected to each other, the charge rapidly recombines, leaving the capacitor uncharged.

12. All connections of capacitors are not simple combinations of series and parallel circuits. As an example of such a complex circuit, consider the network of five capacitors C_1, C_2, C_3, C_4, and C_5 shown below.



This combination cannot be reduced to a simple equivalent by the techniques of combining series and parallel capacitors.

14. The material of the dielectric may be able to withstand a larger electric field than air can withstand before breaking down to pass a spark between the capacitor plates.

16. **(a)** i **(b)** ii

18. **(a)** The equation is only valid when the points *A* and *B* are located in a region where the electric field is uniform (that is, constant in both magnitude and direction). **(b)** No. The field due to a point charge is not a uniform field. **(c)** Yes. The field in the region between a pair of parallel plates is uniform.

Answers to Even Numbered Problems

2. **(a)** -6.0×10^{-4} J **(b)** –50 V

4. -3.20×10^{-19} C

6. 4.3×10^{6} J

8. **(a)** 1.52×10^{5} m/s **(b)** 6.49×10^{6} m/s

10. 40.2 kV

12. 2.2×10^{2} V

14. -9.08 J

16. 8.09×10^{-7} J

18. 7.25×10^{6} m/s

22. **(a)** 48.0 μC **(b)** 6.00 μC

24. **(a)** 800 V **(b)** $Q_f = Q_i/2$

26. 31.0 Å

28. 1.23 kV

30. **(a)** 18.0 μF **(b)** 1.78 μF

32. 3.00 pF and 6.00 pF

34. **(a)** 12.0 μF **(b)** Q_4=144 μC, Q_2=72.0 μC, Q_{24}=Q_8=216 μC

36. Yes. Connect a parallel combination of two capacitors in series with another parallel combination of two capacitors. $\Delta V = 45.0$ V .

38. 30.0 μF

40. 6.04 μF

42. 12.9 μF

44. **(a)** 0.150 J **(b)** 268 V

46. 9.79 kg

48. **(a)** 3.00×10^3 V/m **(b)** 42.5 nC **(c)** 5.31 pC

50. 1.04 m

54. $C_1 = \frac{1}{2}C_p \pm \sqrt{\frac{1}{4}C_p^2 - C_pC_s}$, $C_2 = \frac{1}{2}C_p \mp \sqrt{\frac{1}{4}C_p^2 - C_pC_s}$

56. **(a)** 1.8×10^4 V **(b)** -3.6×10^4 V
(c) -1.8×10^4 V **(d)** -5.4×10^{-2} J

58. **(a)** $C = \frac{ab}{k_e(b-a)}$

60. $\kappa = 2.33$

62. 1.8×10^2 μC on C_1, 89 μC on C_2

64. 121 V

66. **(a)** 0.11 m **(b)** at $x = 4.4$ mm

Problem Solutions

16.1 (a) The work done is $W = F \cdot s\cos\theta = (qE)\cdot s\cos\theta$, or

$$W = (1.60\times10^{-19}\ \text{C})(200\ \text{N/C})(2.00\times10^{-2}\ \text{m})\cos 0° = \boxed{6.40\times10^{-19}\ \text{J}}$$

(b) The change in the electrical potential energy is

$$\Delta PE_e = -W = \boxed{-6.40\times10^{-19}\ \text{J}}$$

(c) The change in the electrical potential is

$$\Delta V = \frac{\Delta PE_e}{q} = \frac{-6.40\times10^{-19}\ \text{J}}{1.60\times10^{-19}\ \text{C}} = \boxed{-4.00\ \text{V}}$$

16.2 (a) We follow the path from (0,0) to (20 cm,0) to (20 cm,50 cm). The work done on the charge by the field is

$$W = W_1 + W_2 = (qE)\cdot s_1\cos\theta_1 + (qE)\cdot s_2\cos\theta_2$$

$$= (qE)\left[(0.20\ \text{m})\cos 0° + (0.50\ \text{m})\cos 90°\right]$$

$$= (12\times10^{-6}\ \text{C})(250\ \text{V/m})\left[(0.20\ \text{m}) + 0\right] = 6.0\times10^{-4}\ \text{J}$$

Thus, $\Delta PE_e = -W = \boxed{-6.0\times10^{-4}\ \text{J}}$

(b) $\Delta V = \dfrac{\Delta PE_e}{q} = \dfrac{-6.0\times10^{-4}\ \text{J}}{12\times10^{-6}\ \text{C}} = -50\ \text{J/C} = \boxed{-50\ \text{V}}$

16.3 The work done by the agent moving the charge out of the cell is

$$W_{input} = -W_{field} = -(-\Delta PE_e) = +q(\Delta V)$$

$$= (1.60\times10^{-19}\ \text{C})\left(+90\times10^{-3}\ \frac{\text{J}}{\text{C}}\right) = \boxed{1.4\times10^{-20}\ \text{J}}$$

16.4 $\Delta PE_e = q(\Delta V) = q(V_f - V_i)$, so $q = \dfrac{\Delta PE_e}{V_f - V_i} = \dfrac{-1.92\times10^{-17}\ \text{J}}{+60.0\ \text{J/C}} = \boxed{-3.20\times10^{-19}\ \text{C}}$

16.5 $E = \dfrac{|\Delta V|}{d} = \dfrac{25\,000\ \text{J/C}}{1.5\times10^{-2}\ \text{m}} = \boxed{1.7\times10^{6}\ \text{N/C}}$

16.6 Since potential difference is work per unit charge $\Delta V = \dfrac{W}{q}$, the work done is

$$W = q(\Delta V) = \left(3.6\times10^{5}\ \text{C}\right)\left(+12\ \text{J/C}\right) = \boxed{4.3\times10^{6}\ \text{J}}$$

16.7 (a) $E = \dfrac{|\Delta V|}{d} = \dfrac{600\ \text{J/C}}{5.33\times10^{-3}\ \text{m}} = \boxed{1.13\times10^{5}\ \text{N/C}}$

(b) $F = |q|E = \left(1.60\times10^{-19}\ \text{C}\right)\left(1.13\times10^{5}\ \text{N/C}\right) = \boxed{1.80\times10^{-14}\ \text{N}}$

(c) $W = F\cdot s\cos\theta$

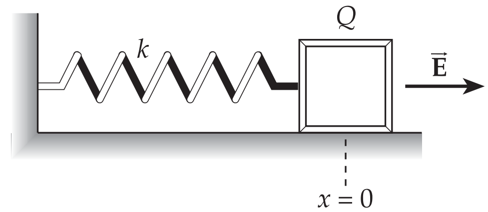
$$= \left(1.80\times10^{-14}\ \text{N}\right)\left[(5.33-2.90)\times10^{-3}\ \text{m}\right]\cos 0^\circ = \boxed{4.38\times10^{-17}\ \text{J}}$$

16.8 From conservation of energy, $\dfrac{1}{2}mv_f^2 - 0 = |q(\Delta V)|$ or $v_f = \sqrt{\dfrac{2|q(\Delta V)|}{m}}$

(a) For the proton, $v_f = \sqrt{\dfrac{2\left|\left(1.60\times10^{-19}\ \text{C}\right)\left(-120\ \text{V}\right)\right|}{1.67\times10^{-27}\ \text{kg}}} = \boxed{1.52\times10^{5}\ \text{m/s}}$

(b) For the electron, $v_f = \sqrt{\dfrac{2\left|\left(-1.60\times10^{-19}\ \text{C}\right)\left(+120\ \text{V}\right)\right|}{9.11\times10^{-31}\ \text{kg}}} = \boxed{6.49\times10^{6}\ \text{m/s}}$

16.9 (a) Use conservation of energy



$$\left(KE + PE_s + PE_e\right)_f = \left(KE + PE_s + PE_e\right)_i$$

or $\Delta(KE) + \Delta(PE_s) + \Delta(PE_e) = 0$

$\Delta(KE) = 0$ since the block is at rest at both beginning and end.

$$\Delta(PE_s) = \frac{1}{2}kx_{\text{max}}^2 - 0,$$

where x_{max} is the maximum stretch of the spring.

$$\Delta(PE_e) = -W = -(QE)x_{\text{max}}$$

Thus, $0 + \frac{1}{2}kx_{\text{max}}^2 - (QE)x_{\text{max}} = 0$, giving

$$x_{\text{max}} = \frac{2QE}{k} = \frac{2\left(50.0\times 10^{-6}\ \text{C}\right)\left(5.00\times 10^{5}\ \text{V/m}\right)}{100\ \text{N/m}} = \boxed{0.500\ \text{m}}$$

(b) At equilibrium, $\Sigma F = -F_s + F_e = 0$, or $-kx_{eq} + QE = 0$

Therefore, $x_{eq} = \frac{QE}{k} = \frac{1}{2}x_{max} = \boxed{0.250\ \text{m}}$

Note that when the block is released from rest, it overshoots the equilibrium position and oscillates with simple harmonic motion in the electric field.

16.10 Using $\Delta y = v_{0y}t + \frac{1}{2}a_y t^2$ for the full flight gives

$$0 = v_{0y}t + \frac{1}{2}a_y t^2, \text{ or } a_y = \frac{-2v_{0y}}{t}$$

Then, using $v_y^2 = v_{0y}^2 + 2a_y(\Delta y)$ for the upward part of the flight gives

$$(\Delta y)_{\text{max}} = \frac{0 - v_{0y}^2}{2a_y} = \frac{-v_{0y}^2}{2\left(-2v_{0y}/t\right)} = \frac{v_{0y}\,t}{4} = \frac{(20.1\ \text{m/s})(4.10\ \text{s})}{4} = 20.6\ \text{m}$$

From Newton's second law, $a_y = \frac{\Sigma F_y}{m} = \frac{-mg - qE}{m} = -\left(g + \frac{qE}{m}\right)$. Equating this to the earlier result gives $a_y = -\left(g + \frac{qE}{m}\right) = \frac{-2v_{0y}}{t}$, so the electric field strength is

$$E = \left(\frac{m}{q}\right)\left[\frac{2v_{0y}}{t} - g\right] = \left(\frac{2.00\text{ kg}}{5.00\times10^{-6}\text{ C}}\right)\left[\frac{2(20.1\text{ m/s})}{4.10\text{ s}} - 9.80\text{ m/s}^2\right] = 1.95\times10^{3}\text{ N/C}$$

Thus, $(\Delta V)_{\max} = (\Delta y_{\max})E = (20.6\text{ m})(1.95\times10^{3}\text{ N/C}) = 4.02\times10^{4}\text{ V} = \boxed{40.2\text{ kV}}$

16.11 (a) $V = \frac{k_e q}{r} = \frac{(8.99\times10^{9}\text{ N}\cdot\text{m}^2/\text{C}^2)(1.60\times10^{-19}\text{ C})}{1.00\times10^{-2}\text{ m}} = \boxed{1.44\times10^{-7}\text{ V}}$

(b) $\Delta V = V_2 - V_1 = \frac{k_e q}{r_2} - \frac{k_e q}{r_1} = (k_e q)\left(\frac{1}{r_2} - \frac{1}{r_1}\right)$

$$= (8.99\times10^{9}\text{ N}\cdot\text{m}^2/\text{C}^2)(1.60\times10^{-19}\text{ C})\left(\frac{1}{0.020\,0\text{ m}} - \frac{1}{0.010\,0\text{ m}}\right)$$

$$= \boxed{-7.19\times10^{-8}\text{ V}}$$

16.12 $V = V_1 + V_2 = k_e\left(\frac{q_1}{r_1} + \frac{q_2}{r_2}\right)$ where $r_1 = 0.60\text{ m} - 0 = 0.60\text{ m}$, and $r_2 = 0.60\text{ m} - 0.30\text{ m} = 0.30\text{ m}$. Thus,

$$V = \left(8.99\times10^{9}\ \frac{\text{N}\cdot\text{m}^2}{\text{C}^2}\right)\left(\frac{3.0\times10^{-9}\text{ C}}{0.60\text{ m}} + \frac{6.0\times10^{-9}\text{ C}}{0.30\text{ m}}\right) = \boxed{2.2\times10^{2}\text{ V}}$$

16.13 (a) Calling the $2.00\ \mu\text{C}$ charge q_3,

$$V = \sum_i \frac{k_e q_i}{r_i} = k_e\left(\frac{q_1}{r_1} + \frac{q_2}{r_2} + \frac{q_3}{\sqrt{r_1^2 + r_2^2}}\right)$$

$$= \left(8.99\times10^9\ \frac{\text{N}\cdot\text{m}^2}{\text{C}^2}\right)\left(\frac{8.00\times10^{-6}\ \text{C}}{0.060\,0\ \text{m}} + \frac{4.00\times10^{-6}\ \text{C}}{0.030\,0\ \text{m}} + \frac{2.00\times10^{-6}\ \text{C}}{\sqrt{(0.060\,0)^2 + (0.030\,0)^2}\ \text{m}}\right)$$

$$V = \boxed{2.67\times10^6\ \text{V}}$$

(b) Replacing $2.00\times10^{-6}\ \text{C}$ by $-2.00\times10^{-6}\ \text{C}$ in part (a) yields

$$V = \boxed{2.13\times10^6\ \text{V}}$$

16.14 $W = q(\Delta V) = q(V_f - V_i)$, and

$V_f = 0$ since the $8.00\ \mu\text{C}$ is infinite distance from other charges.

$$V_i = k_e\left(\frac{q_1}{r_1} + \frac{q_2}{r_2}\right) = \left(8.99\times10^9\ \frac{\text{N}\cdot\text{m}^2}{\text{C}^2}\right)\left(\frac{2.00\times10^{-6}\ \text{C}}{0.030\,0\ \text{m}} + \frac{4.00\times10^{-6}\ \text{C}}{\sqrt{(0.030\,0)^2 + (0.060\,0)^2}\ \text{m}}\right)$$

$$= 1.135\times10^6\ \text{V}$$

Thus, $W = (8.00\times10^{-6}\ \text{C})(0 - 1.135\times10^6\ \text{V}) = \boxed{-9.08\ \text{J}}$

16.15 (a) $V = \sum_i \frac{k_e q_i}{r_i}$

$$= \left(8.99\times10^9\ \frac{\text{N}\cdot\text{m}^2}{\text{C}^2}\right)\left(\frac{5.00\times10^{-9}\ \text{C}}{0.175\ \text{m}} - \frac{3.00\times10^{-9}\ \text{C}}{0.175\ \text{m}}\right) = \boxed{103\ \text{V}}$$

(b) $PE = \dfrac{k_e q_i q_2}{r_{12}}$

$$= \left(8.99\times10^{9}\ \frac{\mathrm{N\cdot m^2}}{\mathrm{C^2}}\right)\frac{\left(5.00\times10^{-9}\ \mathrm{C}\right)\left(-3.00\times10^{-9}\ \mathrm{C}\right)}{0.350\ \mathrm{m}} = \boxed{-3.85\times10^{-7}\ \mathrm{J}}$$

The negative sign means that $\boxed{\text{positive work must be done}}$ to separate the charges (that is, bring them up to a state of zero potential energy).

16.16 The potential at distance $r = 0.300\ \mathrm{m}$ from a charge $Q = +9.00\times10^{-9}\ \mathrm{C}$ is

$$V = \frac{k_e Q}{r} = \frac{\left(8.99\times10^{9}\ \mathrm{N\cdot m^2/C^2}\right)\left(9.00\times10^{-9}\ \mathrm{C}\right)}{0.300\ \mathrm{m}} = +270\ \mathrm{V}$$

Thus, the work required to carry a charge $q = 3.00\times10^{-9}\ \mathrm{C}$ from infinity to this location is

$$W = qV = \left(3.00\times10^{-9}\ \mathrm{C}\right)\left(+270\ \mathrm{V}\right) = \boxed{8.09\times10^{-7}\ \mathrm{J}}$$

16.17 The Pythagorean theorem gives the distance from the midpoint of the base to the charge at the apex of the triangle as

$$r_3 = \sqrt{(4.00\ \mathrm{cm})^2 - (1.00\ \mathrm{cm})^2} = \sqrt{15}\ \mathrm{cm} = \sqrt{15}\times10^{-2}\ \mathrm{m}$$

Then, the potential at the midpoint of the base is $V = \sum_i k_e q_i / r_i$, or

$$V = \left(8.99\times10^{9}\ \frac{\mathrm{N\cdot m^2}}{\mathrm{C^2}}\right)\left(\frac{\left(-7.00\times10^{-9}\ \mathrm{C}\right)}{0.010\,0\ \mathrm{m}} + \frac{\left(-7.00\times10^{-9}\ \mathrm{C}\right)}{0.010\,0\ \mathrm{m}} + \frac{\left(+7.00\times10^{-9}\ \mathrm{C}\right)}{\sqrt{15}\times10^{-2}\ \mathrm{m}}\right)$$

$$= -1.10\times10^{4}\ \mathrm{V} = \boxed{-11.0\ \mathrm{kV}}$$

16.18 Outside the spherical charge distribution, the potential is the same as for a point charge at the center of the sphere,

$$V = k_e Q/r \text{, where } Q = 1.00\times10^{-9}\ \text{C}$$

Thus, $\Delta(PE_e) = q(\Delta V) = -ek_eQ\left(\frac{1}{r_f} - \frac{1}{r_i}\right)$

and from conservation of energy $\Delta(KE) = -\Delta(PE_e)$,

or $\quad \frac{1}{2}m_e v^2 - 0 = -\left[-ek_eQ\left(\frac{1}{r_f} - \frac{1}{r_i}\right)\right]$ This gives $v = \sqrt{\frac{2k_eQe}{m_e}\left(\frac{1}{r_f} - \frac{1}{r_i}\right)}$, or

$$v = \sqrt{\frac{2\left(8.99\times10^9\ \frac{\text{N}\cdot\text{m}^2}{\text{C}^2}\right)\left(1.00\times10^{-9}\ \text{C}\right)\left(1.60\times10^{-19}\ \text{C}\right)}{9.11\times10^{-31}\ \text{kg}}\left(\frac{1}{0.020\,0\ \text{m}} - \frac{1}{0.030\,0\ \text{m}}\right)}$$

$$v = \boxed{7.25\times10^6\ \text{m/s}}$$

16.19 From conservation of energy, $(KE + PE_e)_f = (KE + PE_e)_i$, which gives

$$0 + \frac{k_eQq}{r_f} = \frac{1}{2}m_\alpha v_i^2 + 0 \text{ or } r_f = \frac{2k_eQq}{m_\alpha v_i^2} = \frac{2k_e(79e)(2e)}{m_\alpha v_i^2}$$

$$r_f = \frac{2\left(8.99\times10^9\ \frac{\text{N}\cdot\text{m}^2}{\text{C}^2}\right)(158)\left(1.60\times10^{-19}\ \text{C}\right)^2}{\left(6.64\times10^{-27}\ \text{kg}\right)\left(2.00\times10^7\ \text{m/s}\right)^2} = \boxed{2.74\times10^{-14}\ \text{m}}$$

16.20 By definition, the work required to move a charge from one point to any other point on an equipotential surface is zero. From the definition of work, $W = (F\cos\theta)\cdot s$, the work is zero only if $s = 0$ or $F\cos\theta = 0$. The displacement s cannot be assumed to be zero in all cases. Thus, one must require that $F\cos\theta = 0$. The force F is given by $F = qE$ and neither the charge q nor the field strength E can be assumed to be zero in all cases. Therefore, the only way the work can be zero in all cases is if $\cos\theta = 0$. But if $\cos\theta = 0$, then $\theta = 90°$ or the force (and hence the electric field) must be perpendicular to the displacement **s** (which is tangent to the surface). That is, the field must be perpendicular to the equipotential surface at all points on that surface.

16.21 $V = \frac{k_e Q}{r}$ so

$$r = \frac{k_e Q}{V} = \frac{\left(8.99\times10^{9}\ \text{N}\cdot\text{m}^2/\text{C}^2\right)\left(8.00\times10^{-9}\ \text{C}\right)}{V} = \frac{71.9\ \text{V}\cdot\text{m}}{V}$$

For $V = 100$ V, 50.0 V, and 25.0 V, $\boxed{r = 0.719\ \text{m, 1.44 m, and 2.88 m}}$

The radii are $\boxed{\text{inversely proportional}}$ to the potential.

16.22 (a) $Q = C(\Delta V) = \left(4.00\times10^{-6}\ \text{F}\right)(12.0\ \text{V}) = 48.0\times10^{-6}\ \text{C} = \boxed{48.0\ \mu\text{C}}$

(b) $Q = C(\Delta V) = \left(4.00\times10^{-6}\ \text{F}\right)(1.50\ \text{V}) = 6.00\times10^{-6}\ \text{C} = \boxed{6.00\ \mu\text{C}}$

16.23 (a) $C = \epsilon_0 \frac{A}{d} = \left(8.85\times10^{-12}\ \frac{\text{C}^2}{\text{N}\cdot\text{m}^2}\right)\frac{\left(1.0\times10^{6}\ \text{m}^2\right)}{(800\ \text{m})} = \boxed{1.1\times10^{-8}\ \text{F}}$

(b) $Q_{max} = C(\Delta V)_{max} = C\left(E_{max} d\right)$

$$= \left(1.11\times10^{-8}\ \text{F}\right)\left(3.0\times10^{6}\ \text{N/C}\right)(800\ \text{m}) = \boxed{27\ \text{C}}$$

16.24 For a parallel plate capacitor, $\Delta V = \frac{Q}{C} = \frac{Q}{\epsilon_0 (A/d)} = \frac{Qd}{\epsilon_0 A}$.

(a) Doubling d while holding Q and A constant doubles ΔV to $\boxed{800\ \text{V}}$.

(b) $Q = \frac{(\epsilon_0 A)\Delta V}{d}$ Thus, doubling d while holding ΔV and A constant will cut the charge in half, or $\boxed{Q_f = Q_i/2}$

16.25 (a) $E = \frac{\Delta V}{d} = \frac{20.0\ \text{V}}{1.80\times10^{-3}\ \text{m}} = 1.11\times10^{4}\ \text{V/m} = \boxed{11.1\ \text{kV/m}}$ directed toward the negative plate

(b) $C = \frac{\epsilon_0 A}{d} = \frac{\left(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2\right)\left(7.60\times10^{-4}\ \text{m}^2\right)}{1.80\times10^{-3}\ \text{m}}$

$$= 3.74\times10^{-12}\ \text{F} = \boxed{3.74\ \text{pF}}$$

(c) $Q = C(\Delta V) = (3.74\times10^{-12}\text{ F})(20.0\text{ V}) = 7.47\times10^{-11}\text{ C} = \boxed{74.7\text{ pC}}$ on one plate and $\boxed{-74.7\text{ pC}}$ on the other plate.

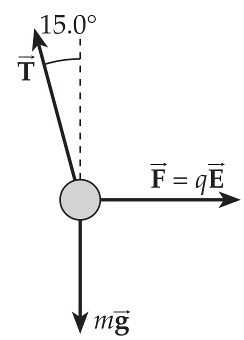
16.26 $C = \dfrac{\epsilon_0 A}{d}$, so $d = \dfrac{\epsilon_0 A}{C} = \dfrac{(8.85\times10^{-12}\text{ C}^2/\text{N}\cdot\text{m}^2)(21.0\times10^{-12}\text{ m}^2)}{60.0\times10^{-15}\text{ F}} = 3.10\times10^{-9}\text{ m}$

$$d = (3.10\times10^{-9}\text{ m})\left(\frac{1\text{ Å}}{10^{-10}\text{ m}}\right) = \boxed{31.0\text{ Å}}$$

16.27 (a) $\Delta V = \dfrac{Q}{C} = \dfrac{Q}{\epsilon_0 A/d} = \dfrac{Qd}{\epsilon_0 A} = \dfrac{(400\times10^{-12}\text{ C})(1.00\times10^{-3}\text{ m})}{(8.85\times10^{-12}\text{ C}^2/\text{N}\cdot\text{m}^2)(5.00\times10^{-4}\text{ m}^2)} = \boxed{90.4\text{ V}}$

(b) $E = \dfrac{|\Delta V|}{d} = \dfrac{90.4\text{ V}}{1.00\times10^{-3}\text{ m}} = \boxed{9.04\times10^{4}\text{ V/m}}$

16.28 $\Sigma F_y = 0 \Rightarrow T\cos15.0° = mg$ or $T = \dfrac{mg}{\cos15.0°}$



$\Sigma F_x = 0 \Rightarrow qE = T\sin15.0° = mg\tan15.0°$

or $E = \dfrac{mg\tan15.0°}{q}$

$$\Delta V = Ed = \frac{mgd\tan15.0°}{q}$$

$$\Delta V = \frac{(350\times10^{-6}\text{ kg})(9.80\text{ m/s}^2)(0.040\,0\text{ m})\tan15.0°}{30.0\times10^{-9}\text{ C}} = 1.23\times10^{3}\text{ V} = \boxed{1.23\text{ kV}}$$

16.29 (a) For series connection, $\dfrac{1}{C_{eq}} = \dfrac{1}{C_1} + \dfrac{1}{C_2} \Rightarrow C_{eq} = \dfrac{C_1C_2}{C_1+C_2}$

$$Q = C_{eq}(\Delta V) = \left(\frac{C_1C_2}{C_1+C_2}\right)\Delta V$$

$$= \left[\frac{(0.050\ \mu\text{F})(0.100\ \mu\text{F})}{0.050\ \mu\text{F} + 0.100\ \mu\text{F}}\right](400\text{ V}) = \boxed{13.3\ \mu\text{C on each}}$$

(b) $Q_1 = C_1(\Delta V) = (0.050\ \mu\text{F})(400\ \text{V}) = \boxed{20.0\ \mu\text{C}}$

$Q_2 = C_2(\Delta V) = (0.100\ \mu\text{F})(400\ \text{V}) = \boxed{40.0\ \mu\text{C}}$

16.30 (a) For parallel connection,

$$C_{eq} = C_1 + C_2 + C_3 = (5.00 + 4.00 + 9.00)\ \mu\text{F} = \boxed{18.0\ \mu\text{F}}$$

(b) For series connection, $\dfrac{1}{C_{eq}} = \dfrac{1}{C_1} + \dfrac{1}{C_2} + \dfrac{1}{C_3}$

$$\frac{1}{C_{eq}} = \frac{1}{5.00\ \mu\text{F}} + \frac{1}{4.00\ \mu\text{F}} + \frac{1}{9.00\ \mu\text{F}}, \text{ giving } C_{eq} = \boxed{1.78\ \mu\text{F}}$$

16.31 (a) Using the rules for combining capacitors in series and in parallel, the circuit is reduced in steps as shown below. The equivalent capacitor is shown to be a $\boxed{2.00\ \mu\text{F}}$ capacitor.

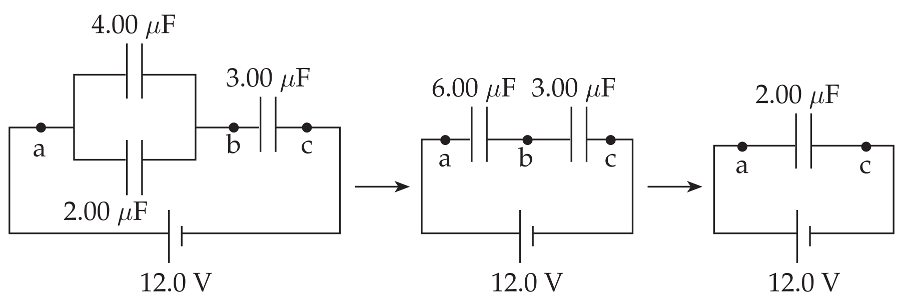


Figure 1 **Figure 2** **Figure 3**

(b) From Figure 3: $Q_{ac} = C_{ac}(\Delta V)_{ac} = (2.00\ \mu\text{F})(12.0\ \text{V}) = 24.0\ \mu\text{C}$

From Figure 2: $Q_{ab} = Q_{bc} = Q_{ac} = 24.0\ \mu\text{C}$

Thus, the charge on the $3.00\ \mu\text{F}$ capacitor is $Q_3 = \boxed{24.0\ \mu\text{C}}$

Continuing to use Figure 2, $(\Delta V)_{ab} = \dfrac{Q_{ab}}{C_{ab}} = \dfrac{24.0\ \mu\text{C}}{6.00\ \mu\text{F}} = 4.00\ \text{V}$

and $(\Delta V)_3 = (\Delta V)_{bc} = \dfrac{Q_{bc}}{C_{bc}} = \dfrac{24.0\ \mu\text{C}}{3.00\ \mu\text{F}} = \boxed{8.00\ \text{V}}$

From Figure 1, $(\Delta V)_4 = (\Delta V)_2 = (\Delta V)_{ab} = \boxed{4.00\ \text{V}}$

and $Q_4 = C_4(\Delta V)_4 = (4.00\ \mu\text{F})(4.00\ \text{V}) = \boxed{16.0\ \mu\text{C}}$

$Q_2 = C_2(\Delta V)_2 = (2.00\ \mu\text{F})(4.00\ \text{V}) = \boxed{8.00\ \mu\text{C}}$

16.32 $C_{parallel} = C_1 + C_2 = 9.00\ \text{pF} \Rightarrow C_1 = 9.00\ \text{pF} - C_2$ (1)

$$\frac{1}{C_{series}} = \frac{1}{C_1} + \frac{1}{C_2} \Rightarrow C_{series} = \frac{C_1 C_2}{C_1 + C_2} = 2.00\ \text{pF}$$

Thus, using equation (1), $C_{series} = \dfrac{(9.00\ \text{pF} - C_2)C_2}{(9.00\ \text{pF} - C_2) + C_2} = 2.00\ \text{pF}$ which reduces to

$$C_2^2 - (9.00\ \text{pF})C_2 + 18.0\ (\text{pF})^2 = 0\text{, or } (C_2 - 6.00\ \text{pF})(C_2 - 3.00\ \text{pF}) = 0$$

Therefore, either $C_2 = 6.00\ \text{pF}$ and, from equation (1), $C_1 = 3.00\ \text{pF}$

or $C_2 = 3.00\ \text{pF}$ and $C_1 = 6.00\ \text{pF}$.

We conclude that the two capacitances are $\boxed{3.00\ \text{pF and } 6.00\ \text{pF}}$.

16.33

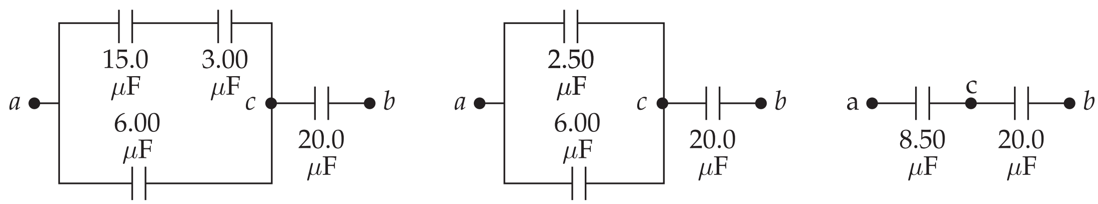


Figure 1 **Figure 2** **Figure 3**

(a) The equivalent capacitance of the upper branch between points *a* and *c* in Figure 1 is

$$C_s = \frac{(15.0\ \mu\text{F})(3.00\ \mu\text{F})}{15.0\ \mu\text{F} + 3.00\ \mu\text{F}} = 2.50\ \mu\text{F}$$

Then, using Figure 2, the total capacitance between points *a* and *c* is

$$C_{ac} = 2.50\ \mu\text{F}+6.00\ \mu\text{F}=8.50\ \mu\text{F}$$

From Figure 3, the total capacitance is

$$C_{eq} = \left(\frac{1}{8.50\ \mu\text{F}} + \frac{1}{20.0\ \mu\text{F}}\right)^{-1} = \boxed{5.96\ \mu\text{F}}$$

(b) $$Q_{ab} = Q_{ac} = Q_{cb} = (\Delta V)_{ab} C_{eq}$$

$$= (15.0\ \text{V})(5.96\ \mu\text{F}) = 89.5\ \mu\text{C}$$

Thus, the charge on the $20.0\ \mu\text{C}$ is $Q_{20} = Q_{cb} = \boxed{89.5\ \mu\text{C}}$

$$(\Delta V)_{ac} = (\Delta V)_{ab} - (\Delta V)_{bc} = 15.0\ \text{V} - \left(\frac{89.5\ \mu\text{C}}{20.0\ \mu\text{F}}\right) = 10.53\ \text{V}$$

Then, $Q_6 = (\Delta V)_{ac}(6.00\ \mu F) = \boxed{63.2\ \mu\text{C}}$ and

$$Q_{15} = Q_3 = (\Delta V)_{ac}(2.50\ \mu F) = \boxed{26.3\ \mu\text{C}}$$

16.34 (a) The combination reduces to an equivalent capacitance of $\boxed{12.0\ \mu\text{F}}$ in stages as shown below.

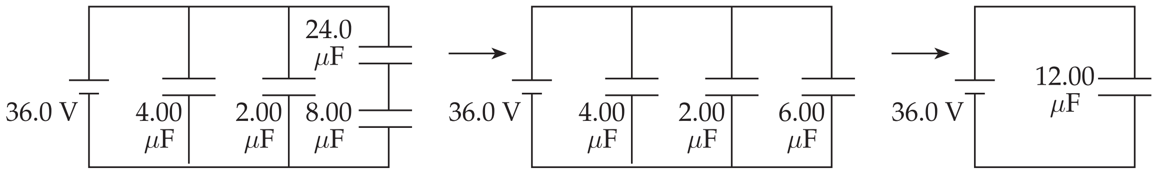


Figure 1 **Figure 2** **Figure 3**

(b) From Figure 2, $Q_4 = (4.00\ \mu\text{F})(36.0\ \text{V}) = \boxed{144\ \mu\text{C}}$

$Q_2 = (2.00\ \mu\text{F})(36.0\ \text{V}) = \boxed{72.0\ \mu\text{C}}$

and $Q_6 = (6.00\ \mu\text{F})(36.0\ \text{V}) = 216\ \mu\text{C}$

Then, from Figure 1, $Q_{24} = Q_8 = Q_6 = \boxed{216\ \mu\text{C}}$

16.35

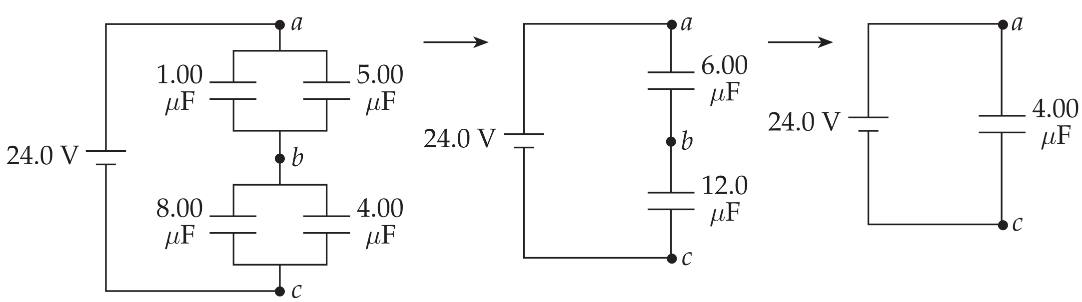


Figure 1 **Figure 2** **Figure 3**

The circuit may be reduced in steps as shown above.

Using the Figure 3, $Q_{ac} = (4.00\ \mu\text{F})(24.0\ \text{V}) = 96.0\ \mu\text{C}$

Then, in Figure 2, $(\Delta V)_{ab} = \dfrac{Q_{ac}}{C_{ab}} = \dfrac{96.0\ \mu\text{C}}{6.00\ \mu\text{F}} = 16.0\ \text{V}$

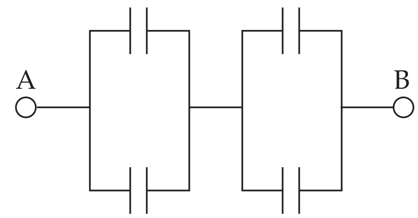
and $(\Delta V)_{bc} = (\Delta V)_{ac} - (\Delta V)_{ab} = 24.0\ \text{V} - 16.0\ \text{V} = 8.00\ \text{V}$

Finally, using Figure 1, $Q_1 = C_1(\Delta V)_{ab} = (1.00\ \mu\text{F})(16.0\ \text{V}) = \boxed{16.0\ \mu\text{C}}$

$$Q_5 = (5.00\ \mu\text{F})(\Delta V)_{ab} = \boxed{80.0\ \mu\text{C}}, \quad Q_8 = (8.00\ \mu\text{F})(\Delta V)_{bc} = \boxed{64.0\ \mu\text{C}}$$

and $Q_4 = (4.00\ \mu\text{F})(\Delta V)_{bc} = \boxed{32.0\ \mu\text{C}}$

16.36 The technician combines two of the capacitors in parallel making a capacitor of capacitance $200\ \mu\text{F}$. Then she does it again with two more of the capacitors. Then the two resulting $200\ \mu\text{F}$ capacitors are connected in series to yield an equivalent capacitance of $100\ \mu\text{F}$. Because of the symmetry of the solution, every capacitor in the combination has the same voltage across it,



$$\Delta V = (\Delta V)_{ab}/2 = (90.0\ \text{V})/2 = \boxed{45.0\ \text{V}}$$

16.37 (a) From $Q = C(\Delta V)$, $Q_{25} = (25.0\ \mu\text{F})(50.0\ \text{V}) = 1.25\times10^3\ \mu\text{C} = \boxed{1.25\ \text{mC}}$

and $Q_{40} = (40.0\ \mu\text{F})(50.0\ \text{V}) = 2.00\times10^3\ \mu\text{C} = \boxed{2.00\ \text{mC}}$

(b) When the two capacitors are connected in parallel, the equivalent capacitance is $C_{eq} = C_1 + C_2 = 25.0\ \mu\text{F}+40.0\ \mu\text{F} = 65.0\ \mu\text{F}$.

Since the negative plate of one was connected to the positive plate of the other, the total charge stored in the parallel combination is

$$Q = Q_{40} - Q_{25} = 2.00\times10^3\ \mu\text{C} - 1.25\times10^3\ \mu\text{C} = 750\ \mu\text{C}$$

The potential difference across each capacitor of the parallel combination is

$$\Delta V = \frac{Q}{C_{eq}} = \frac{750\ \mu\text{C}}{65.0\ \mu\text{F}} = \boxed{11.5\ \text{V}}$$

and the final charge stored in each capacitor is

$$Q'_{25} = C_1(\Delta V) = (25.0\ \mu\text{F})(11.5\ \text{V}) = \boxed{288\ \mu\text{C}}$$

and $Q'_{40} = Q - Q'_{25} = 750\ \mu\text{C} - 288\ \mu\text{C} = \boxed{462\ \mu\text{C}}$

16.38 From $Q = C(\Delta V)$, the initial charge of each capacitor is

$$Q_{10} = (10.0\ \mu\text{F})(12.0\ \text{V}) = 120\ \mu\text{C} \text{ and } Q_x = C_x(0) = 0$$

After the capacitors are connected in parallel, the potential difference across each is $\Delta V' = 3.00\ \text{V}$, and the total charge of $Q = Q_{10} + Q_x = 120\ \mu\text{C}$ is divided between the two capacitors as

$$Q'_{10} = (10.0\ \mu\text{F})(3.00\ \text{V}) = 30.0\ \mu\text{C} \text{ and}$$

$$Q'_x = Q - Q'_{10} = 120\ \mu\text{C} - 30.0\ \mu\text{C} = 90.0\ \mu\text{C}$$

Thus, $C_x = \dfrac{Q'_x}{\Delta V'} = \dfrac{90.0\ \mu\text{C}}{3.00\ \text{V}} = \boxed{30.0\ \mu\text{F}}$

16.39 From $Q = C(\Delta V)$, the initial charge of each capacitor is

$$Q_1 = (1.00\ \mu\text{F})(10.0\ \text{V}) = 10.0\ \mu\text{C} \text{ and } Q_2 = (2.00\ \mu\text{F})(0) = 0$$

After the capacitors are connected in parallel, the potential difference across one is the same as that across the other. This gives

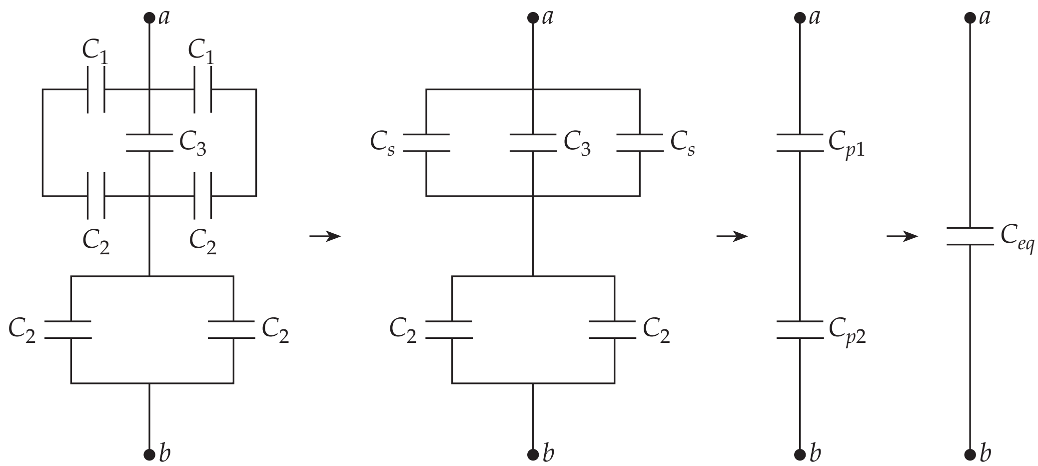
$$\Delta V = \frac{Q'_1}{1.00\ \mu\text{F}} = \frac{Q'_2}{2.00\ \mu\text{F}} \text{ or } Q'_2 = 2Q'_1 \qquad (1)$$

From conservation of charge, $Q'_1 + Q'_2 = Q_1 + Q_2 = 10.0\ \mu\text{C}$. Then, substituting from equation (1), this becomes

$Q'_1 + 2Q'_1 = 10.0\ \mu\text{C}$, giving $\qquad Q'_1 = \boxed{\dfrac{10}{3}\ \mu\text{C}}$

Finally, from equation (1), $\qquad Q'_2 = \boxed{\dfrac{20}{3}\ \mu\text{C}}$

16.40 The original circuit reduces to a single equivalent capacitor in the steps shown below.



$$C_s = \left(\frac{1}{C_1}+\frac{1}{C_2}\right)^{-1} = \left(\frac{1}{5.00\ \mu\text{F}}+\frac{1}{10.0\ \mu\text{F}}\right)^{-1} = 3.33\ \mu\text{F}$$

$$C_{p1} = C_s + C_3 + C_s = 2(3.33\ \mu\text{F}) + 2.00\ \mu\text{F} = 8.66\ \mu\text{F}$$

$$C_{p2} = C_2 + C_2 = 2(10.0\ \mu\text{F}) = 20.0\ \mu\text{F}$$

$$C_{eq} = \left(\frac{1}{C_{p1}}+\frac{1}{C_{p2}}\right)^{-1} = \left(\frac{1}{8.66\ \mu\text{F}}+\frac{1}{20.0\ \mu\text{F}}\right)^{-1} = \boxed{6.04\ \mu\text{F}}$$

16.41 Refer to the solution of Problem 16.40 given above. The total charge stored between points *a* and *b* is

$$Q_{eq} = C_{eq}(\Delta V)_{ab} = (6.04\ \mu\text{F})(60.0\ \text{V}) = 362\ \mu\text{C}$$

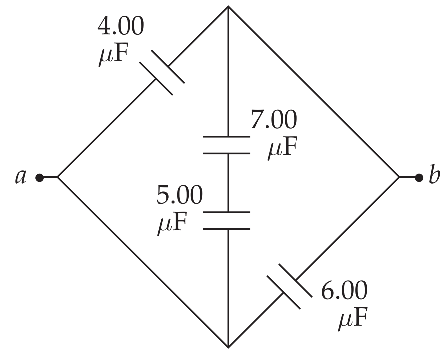
Then, looking at the third figure, observe that the charges of the series capacitors of that figure are $Q_{p1} = Q_{p2} = Q_{eq} = 362\ \mu\text{C}$. Thus, the potential difference across the upper parallel combination shown in the second figure is

$$(\Delta V)_{p1} = \frac{Q_{p1}}{C_{p1}} = \frac{362\ \mu\text{C}}{8.66\ \mu\text{F}} = 41.8\ \text{V}$$

Finally, the charge on C_3 is

$$Q_3 = C_3(\Delta V)_{p1} = (2.00\ \mu\text{F})(41.8\ \text{V}) = \boxed{83.6\ \mu\text{C}}$$

16.42 Recognize that the $7.00\ \mu\text{F}$ and the $5.00\ \mu\text{F}$ of the center branch are connected in series. The total capacitance of that branch is



$$C_s = \left(\frac{1}{5.00} + \frac{1}{7.00}\right)^{-1} = 2.92\ \mu\text{F}$$

Then recognize that this capacitor, the $4.00\ \mu\text{F}$ capacitor, and the $6.00\ \mu\text{F}$ capacitor are all connected in parallel between points *a* and *b*. Thus, the equivalent capacitance between points *a* and *b* is

$$C_{eq} = 4.00\ \mu\text{F} + 2.92\ \mu\text{F} + 6.00\ \mu\text{F} = \boxed{12.9\ \mu\text{F}}$$

16.43 The capacitance is

$$C = \frac{\epsilon_0 A}{d} = \frac{\left(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2\right)\left(2.00\times10^{-4}\ \text{m}^2\right)}{5.00\times10^{-3}\ \text{m}} = 3.54\times10^{-13}\ \text{F}$$

and the stored energy is

$$W = \frac{1}{2}C(\Delta V)^2 = \frac{1}{2}\left(3.54\times10^{-13}\ \text{F}\right)(12.0\ \text{V})^2 = \boxed{2.55\times10^{-11}\ \text{J}}$$

16.44 (a) When connected in parallel, the energy stored is

$$W = \frac{1}{2}C_1(\Delta V)^2 + \frac{1}{2}C_2(\Delta V)^2 = \frac{1}{2}(C_1 + C_2)(\Delta V)^2$$

$$= \frac{1}{2}\left[(25.0 + 5.00)\times10^{-6}\ \text{F}\right](100\ \text{V})^2 = \boxed{0.150\ \text{J}}$$

(b) When connected in series, the equivalent capacitance is

$$C_{eq} = \left(\frac{1}{25.0} + \frac{1}{5.00}\right)^{-1}\ \mu\text{F} = 4.17\ \mu\text{F}$$

From $W = \frac{1}{2}C_{eq}(\Delta V)^2$, the potential difference required to store the same energy as in part (a) above is

$$\Delta V = \sqrt{\frac{2W}{C_{eq}}} = \sqrt{\frac{2(0.150\ \text{J})}{4.17\times10^{-6}\ \text{F}}} = \boxed{268\ \text{V}}$$

16.45 The capacitance of this parallel plate capacitor is

$$C = \epsilon_0 \frac{A}{d} = \left(8.85\times10^{-12}\ \frac{\text{C}^2}{\text{N}\cdot\text{m}^2}\right)\frac{\left(1.0\times10^6\ \text{m}^2\right)}{(800\ \text{m})} = 1.1\times10^{-8}\ \text{F}$$

With an electric field strength of $E = 3.0\times10^6\ \text{N/C}$ and a plate separation of $d = 800\ \text{m}$, the potential difference between plates is

$$\Delta V = Ed = \left(3.0\times10^6\ \text{V/m}\right)(800\ \text{m}) = 2.4\times10^9\ \text{V}$$

Thus, the energy available for release in a lightning strike is

$$W = \frac{1}{2}C(\Delta V)^2 = \frac{1}{2}\left(1.1\times10^{-8}\ \text{F}\right)\left(2.4\times10^9\ \text{V}\right)^2 = \boxed{3.2\times10^{10}\ \text{J}}$$

16.46 The energy transferred to the water is

$$W = \frac{1}{100}\left[\frac{1}{2}Q(\Delta V)\right] = \frac{(50.0\ \text{C})\left(1.00\times10^8\ \text{V}\right)}{200} = 2.50\times10^7\ \text{J}$$

Thus, if m is the mass of water boiled away,

$$W = m\left[c(\Delta T) + L_v\right] \text{ becomes}$$

$$2.50\times10^7\ \text{J} = m\left[\left(4186\ \frac{\text{J}}{\text{kg}\cdot{}^\circ\text{C}}\right)(100^\circ\text{C} - 30.0^\circ\text{C}) + 2.26\times10^6\ \text{J/kg}\right]$$

giving $$m = \frac{2.50\times10^7\ \text{J}}{2.55\ \text{J/kg}} = \boxed{9.79\ \text{kg}}$$

16.47 The initial capacitance (with air between the plates) is $C_i = Q/(\Delta V)_i$, and the final capacitance (with the dielectric inserted) is $C_f = Q/(\Delta V)_f$ where Q is the constant quantity of charge stored on the plates.

Thus, the dielectric constant is $\kappa = \dfrac{C_f}{C_i} = \dfrac{(\Delta V)_i}{(\Delta V)_f} = \dfrac{100\ \text{V}}{25\ \text{V}} = \boxed{4.0}$

16.48 (a) $E = \dfrac{\Delta V}{d} = \dfrac{6.00\ \text{V}}{2.00\times10^{-3}\ \text{m}} = \boxed{3.00\times10^3\ \text{V/m}}$

(b) With air between the plates, the capacitance is

$$C_{air} = \epsilon_0 \frac{A}{d} = \left(8.85 \times 10^{-12}\ \frac{C^2}{N \cdot m^2}\right) \frac{\left(2.00 \times 10^{-4}\ m^2\right)}{\left(2.00 \times 10^{-3}\ m\right)} = 8.85 \times 10^{-13}\ F$$

and with water $(\kappa = 80)$ between the plates, the capacitance is

$$C = \kappa C_{air} = (80)\left(8.85 \times 10^{-13}\ F\right) = 7.08 \times 10^{-11}\ F$$

The stored charge when water is between the plates is

$$Q = C(\Delta V) = \left(7.08 \times 10^{-11}\ F\right)(6.00\ V) = 4.25 \times 10^{-10}\ C = \boxed{42.5\ nC}$$

(c) When air is the dielectric between the plates, the stored charge is

$$Q_{air} = C_{air}(\Delta V) = \left(8.85 \times 10^{-13}\ F\right)(6.00\ V) = 5.31 \times 10^{-12}\ C = \boxed{5.31\ pC}$$

16.49 (a) The dielectric constant for Teflon® is $\kappa = 2.1$, so the capacitance is

$$C = \frac{\kappa \epsilon_0 A}{d} = \frac{(2.1)\left(8.85 \times 10^{-12}\ C^2/N \cdot m^2\right)\left(175 \times 10^{-4}\ m^2\right)}{0.040\,0 \times 10^{-3}\ m}$$

$$C = 8.13 \times 10^{-9}\ F = \boxed{8.13\ nF}$$

(b) For Teflon®, the dielectric strength is $E_{max} = 60.0 \times 10^6\ V/m$, so the maximum voltage is

$$V_{max} = E_{max} d = \left(60.0 \times 10^6\ V/m\right)\left(0.040\,0 \times 10^{-3}\ m\right)$$

$$V_{max} = 2.40 \times 10^3\ V = \boxed{2.40\ kV}$$

16.50 Before the capacitor is rolled, the capacitance of this parallel plate capacitor is

$$C = \frac{\kappa \epsilon_0 A}{d} = \frac{\kappa \epsilon_0 (w \times L)}{d}$$

where A is the surface area of one side of a foil strip. Thus, the required length is

$$L = \frac{C \cdot d}{\kappa \epsilon_0 w} = \frac{\left(9.50 \times 10^{-8}\ F\right)\left(0.025\,0 \times 10^{-3}\ m\right)}{(3.70)\left(8.85 \times 10^{-12}\ C^2/N \cdot m^2\right)\left(7.00 \times 10^{-2}\ m\right)} = \boxed{1.04\ m}$$

16.51 (a) $V = \frac{m}{\rho} = \frac{1.00\times10^{-12}\ \text{kg}}{1100\ \text{kg/m}^3} = \boxed{9.09\times10^{-16}\ \text{m}^3}$

Since $V = \frac{4\pi r^3}{3}$, the radius is $r = \left[\frac{3V}{4\pi}\right]^{1/3}$, and the surface area is

$$A = 4\pi r^2 = 4\pi\left[\frac{3V}{4\pi}\right]^{2/3} = 4\pi\left[\frac{3\left(9.09\times10^{-16}\ \text{m}^3\right)}{4\pi}\right]^{2/3} = \boxed{4.54\times10^{-10}\ \text{m}^2}$$

(b) $C = \frac{\kappa \in_0 A}{d}$

$$= \frac{(5.00)\left(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2\right)\left(4.54\times10^{-10}\ \text{m}^2\right)}{100\times10^{-9}\ \text{m}} = \boxed{2.01\times10^{-13}\ \text{F}}$$

(c) $Q = C(\Delta V) = \left(2.01\times10^{-13}\ \text{F}\right)\left(100\times10^{-3}\ \text{V}\right) = \boxed{2.01\times10^{-14}\ \text{C}}$

and the number of electronic charges is

$$n = \frac{Q}{e} = \frac{2.01\times10^{-14}\ \text{C}}{1.60\times10^{-19}\ \text{C}} = \boxed{1.26\times10^{5}}$$

16.52 Since the capacitors are in parallel, the equivalent capacitance is

$$C_{eq} = C_1 + C_2 + C_3 = \frac{\in_0 A_1}{d} + \frac{\in_0 A_2}{d} + \frac{\in_0 A_3}{d} = \frac{\in_0 \left(A_1 + A_2 + A_3\right)}{d}$$

or $$C_{eq} = \boxed{\frac{\in_0 A}{d} \text{ where } A = A_1 + A_2 + A_3}$$

16.53 Since the capacitors are in series, the equivalent capacitance is given by

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} = \frac{d_1}{\in_0 A} + \frac{d_2}{\in_0 A} + \frac{d_3}{\in_0 A} = \frac{d_1 + d_2 + d_3}{\in_0 A}$$

or $$C_{eq} = \boxed{\frac{\in_0 A}{d} \text{ where } d = d_1 + d_2 + d_3}$$

16.54 For the parallel combination: $C_p = C_1 + C_2$ which gives $\quad C_2 = C_p - C_1 \quad (1)$

For the series combination: $\dfrac{1}{C_s} = \dfrac{1}{C_1} + \dfrac{1}{C_2}$ or $\dfrac{1}{C_2} = \dfrac{1}{C_s} - \dfrac{1}{C_1} = \dfrac{C_1 - C_s}{C_s C_1}$

Thus, we have $\quad C_2 = \dfrac{C_s C_1}{C_1 - C_s}\quad$ and equating this to Equation (1) above gives

$$C_p - C_1 = \frac{C_s C_1}{C_1 - C_s} \qquad \text{or} \qquad C_p C_1 - C_p C_s - C_1^2 + \cancel{C_s C_1} = \cancel{C_s C_1}$$

We write this result as : $\quad C_1^2 - C_p C_1 + C_p C_s = 0$

and use the quadratic formula to obtain $\quad \boxed{C_1 = \dfrac{1}{2}C_p \pm \sqrt{\dfrac{1}{4}C_p^2 - C_p C_s}}$

Then, Equation (1) gives $\quad \boxed{C_2 = \dfrac{1}{2}C_p \mp \sqrt{\dfrac{1}{4}C_p^2 - C_p C_s}}$

16.55 The charge stored on the capacitor by the battery is

$$Q = C(\Delta V)_1 = C(100 \text{ V})$$

This is also the total charge stored in the parallel combination when this charged capacitor is connected in parallel with an uncharged $10.0\text{-}\mu\text{F}$ capacitor. Thus, if $(\Delta V)_2$ is the resulting voltage across the parallel combination, $Q = C_p(\Delta V)_2$ gives

$$C(100 \text{ V}) = (C + 10.0\ \mu\text{F})(30.0 \text{ V}) \text{ or } (70.0 \text{ V})C = (30.0 \text{ V})(10.0\ \mu\text{F})$$

and $\quad C = \left(\dfrac{30.0 \text{ V}}{70.0 \text{ V}}\right)(10.0\ \mu\text{F}) = \boxed{4.29\ \mu\text{F}}$

16.56 (a) The $1.0\text{-}\mu C$ is located 0.50 m from point *P*, so its contribution to the potential at *P* is

$$V_1 = k_e \frac{q_1}{r_1} = \left(8.99 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2\right)\left(\frac{1.0 \times 10^{-6} \text{ C}}{0.50 \text{ m}}\right) = \boxed{1.8 \times 10^4 \text{ V}}$$

(b) The potential at *P* due to the $-2.0\text{-}\mu C$ charge located 0.50 m away is

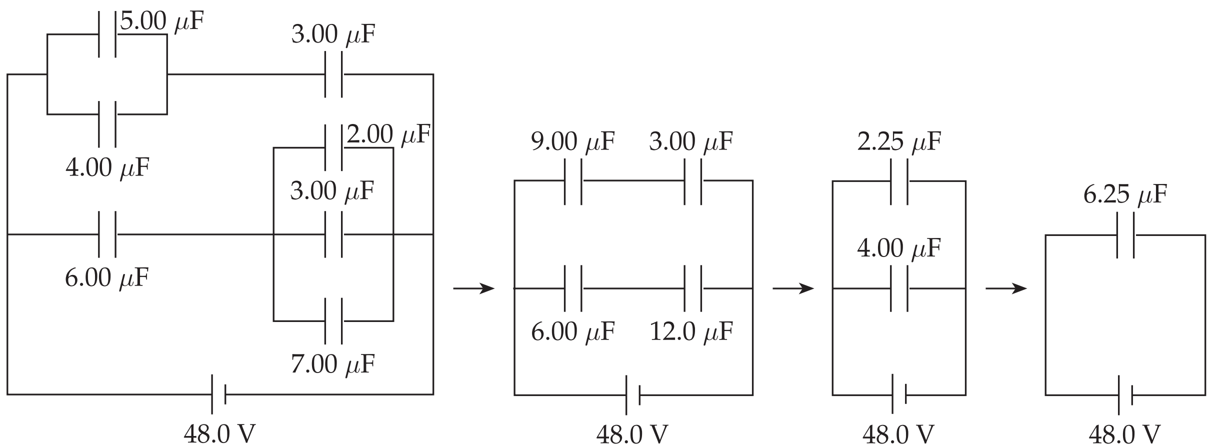
$$V_2 = k_e \frac{q_2}{r_2} = \left(8.99 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2\right)\left(\frac{-2.0 \times 10^{-6} \text{ C}}{0.50 \text{ m}}\right) = \boxed{-3.6 \times 10^4 \text{ V}}$$

(c) The total potential at point P is $V_P = V_1 + V_2 = (+1.8 - 3.6)\times 10^4\ \text{V} = \boxed{-1.8\times 10^4\ \text{V}}$

(d) The work required to move a charge $q = 3.0\ \mu\text{C}$ to point P from infinity is

$$W = q\Delta V = q(V_P - V_\infty) = (3.0\times 10^{-6}\ \text{C})(-1.8\times 10^4\ \text{V} - 0) = \boxed{-5.4\times 10^{-2}\ \text{J}}$$

16.57 The stages for the reduction of this circuit are shown below.



Thus, $C_{eq} = \boxed{6.25\ \mu\text{F}}$

16.58 (a) Due to spherical symmetry, the charge on each of the concentric spherical shells will be uniformly distributed over that shell. Inside a spherical surface having a uniform charge distribution, the electric field due to the charge on that surface is zero. Thus, in this region, the potential due to the charge on that surface is constant and equal to the potential at the surface. Outside a spherical surface having a uniform charge distribution, the potential due to the charge on that surface is given by $V = \dfrac{k_e q}{r}$ where r is the distance from the center of that surface and q is the charge on that surface.

In the region between a pair of concentric spherical shells, with the inner shell having charge $+Q$ and the outer shell having radius b and charge $-Q$, the total electric potential is given by

$$V = V_{\substack{\text{due to}\\ \text{inner shell}}} + V_{\substack{\text{due to}\\ \text{outer shell}}} = \frac{k_e Q}{r} + \frac{k_e(-Q)}{b} = k_e Q\left(\frac{1}{r} - \frac{1}{b}\right)$$

The potential difference between the two shells is therefore,

$$\Delta V = V\big|_{r=a} - V\big|_{r=b} = k_e Q\left(\frac{1}{a}-\frac{1}{b}\right) - k_e Q\left(\frac{1}{b}-\frac{1}{b}\right) = k_e Q\left(\frac{b-a}{ab}\right)$$

The capacitance of this device is given by

$$C = \frac{Q}{\Delta V} = \boxed{\frac{ab}{k_e(b-a)}}$$

(b) When $b >> a$, then $b - a \approx b$. Thus, in the limit as $b \to \infty$, the capacitance found above becomes

$$C \to \frac{ab}{k_e(b)} = \frac{a}{k_e} = \boxed{4\pi \in_0 a}$$

16.59 The energy stored in a charged capacitor is $W = \frac{1}{2}C(\Delta V)^2$. Hence,

$$\Delta V = \sqrt{\frac{2W}{C}} = \sqrt{\frac{2(300 \text{ J})}{30.0\times10^{-6} \text{ F}}} = 4.47\times10^3 \text{ V} = \boxed{4.47 \text{ kV}}$$

16.60 From $Q = C(\Delta V)$, the capacitance of the capacitor with air between the plates is

$$C_0 = \frac{Q_0}{\Delta V} = \frac{150\ \mu\text{C}}{\Delta V}$$

After the dielectric is inserted, the potential difference is held to the original value, but the charge changes to $Q = Q_0 + 200\ \mu\text{C} = 350\ \mu\text{C}$. Thus, the capacitance with the dielectric slab in place is

$$C = \frac{Q}{\Delta V} = \frac{350\ \mu\text{C}}{\Delta V}$$

The dielectric constant of the dielectric slab is therefore

$$\kappa = \frac{C}{C_0} = \left(\frac{350\ \mu\text{C}}{\Delta V}\right)\left(\frac{\Delta V}{150\ \mu\text{C}}\right) = \frac{350}{150} = \boxed{2.33}$$

16.61 The charges initially stored on the capacitors are

$$Q_1 = C_1(\Delta V)_i = (6.0\ \mu\text{F})(250\ \text{V}) = 1.5\times 10^3\ \mu\text{C}$$

and $$Q_2 = C_2(\Delta V)_i = (2.0\ \mu\text{F})(250\ \text{V}) = 5.0\times 10^2\ \mu\text{C}$$

When the capacitors are connected in parallel, with the negative plate of one connected to the positive plate of the other, the net stored charge is

$$Q = Q_1 - Q_2 = 1.5\times 10^3\ \mu\text{C} - 5.0\times 10^2\ \mu\text{C} = 1.0\times 10^3\ \mu\text{C}$$

The equivalent capacitance of the parallel combination is $C_{eq} = C_1 + C_2 = 8.0\ \mu\text{F}$. Thus, the final potential difference across each of the capacitors is

$$(\Delta V)' = \frac{Q}{C_{eq}} = \frac{1.0\times 10^3\ \mu\text{C}}{8.0\ \mu\text{F}} = 125\ \text{V}$$

and the final charge on each capacitor is

$$Q_1' = C_1(\Delta V)' = (6.0\ \mu\text{F})(125\ \text{V}) = 750\ \mu\text{C} = \boxed{0.75\ \text{mC}}$$

and $$Q_2' = C_2(\Delta V)' = (2.0\ \mu\text{F})(125\ \text{V}) = 250\ \mu\text{C} = \boxed{0.25\ \text{mC}}$$

16.62 When connected in series, the equivalent capacitance is

$$C_{eq} = \left(\frac{1}{C_1} + \frac{1}{C_2}\right)^{-1} = \left(\frac{1}{4.0\ \mu\text{F}} + \frac{1}{2.0\ \mu\text{F}}\right)^{-1} = \frac{4}{3}\ \mu\text{F}$$

and the charge stored on each capacitor is

$$Q_1 = Q_2 = Q_{eq} = C_{eq}(\Delta V)_i = \left(\frac{4}{3}\ \mu\text{F}\right)(100\ \text{V}) = \frac{400}{3}\ \mu\text{C}$$

When the capacitors are reconnected in parallel, with the positive plate of one connected to the positive plate of the other, the new equivalent capacitance is $C_{eq}' = C_1 + C_2 = 6.0\ \mu\text{F}$ and the net stored charge is $Q' = Q_1 + Q_2 = 800/3\ \mu\text{C}$. Therefore, the final potential difference across each of the capacitors is

$$(\Delta V)' = \frac{Q'}{C_{eq}'} = \frac{800/3\ \mu\text{C}}{6.0\ \mu\text{F}} = 44.4\ \text{V}$$

The final charge on each of the capacitors is

$$Q_1' = C_1(\Delta V)' = (4.0\ \mu\text{F})(44.4\ \text{V}) = \boxed{1.8\times 10^2\ \mu\text{C}}$$

and $Q_2' = C_2(\Delta V)' = (2.0\ \mu\text{F})(44.4\ \text{V}) = \boxed{89\ \mu\text{C}}$

16.63 (a) $V = V_1 + V_2 + V_3 = \dfrac{k_eQ}{x+d} - \dfrac{2k_eQ}{x} + \dfrac{k_eQ}{x-d}$

$$= k_eQ\left[\frac{x(x-d) - 2(x^2-d^2) + x(x+d)}{x(x^2-d^2)}\right]$$

which simplifies to $V = \dfrac{2k_eQd^2}{x(x^2-d^2)} = \boxed{\dfrac{2k_eQd^2}{x^3 - xd^2}}$

(b) When $x >> d$, then $x^2 - d^2 \approx x^2$

and $V = \dfrac{2k_eQd^2}{x(x^2-d^2)}$ becomes $V \approx \boxed{\dfrac{2k_eQd^2}{x^3}}$

16.64 The energy required to melt the lead sample is

$$\begin{aligned} W &= m\left[c_{Pb}(\Delta T) + L_f\right] \\ &= (6.00\times 10^{-6}\ \text{kg})\left[(128\ \text{J/kg}\cdot{}^\circ\text{C})(327.3^\circ\text{C} - 20.0^\circ\text{C}) + 24.5\times 10^3\ \text{J/kg}\right] \\ &= 0.383\ \text{J} \end{aligned}$$

The energy stored in a capacitor is $W = \frac{1}{2}C(\Delta V)^2$, so the required potential difference is

$$\Delta V = \sqrt{\frac{2W}{C}} = \sqrt{\frac{2(0.383\ \text{J})}{52.0\times 10^{-6}\ \text{F}}} = \boxed{121\ \text{V}}$$

16.65 The capacitance of a parallel plate capacitor is $C = \dfrac{\kappa \in_0 A}{d}$

Thus, $\kappa \in_0 A = C \cdot d$, and the given force equation may be rewritten as

$$F = \frac{Q^2}{2\kappa \in_0 A} = \frac{Q^2}{2C \cdot d} = \frac{(Q/C)^2 C}{2d} = \frac{C(\Delta V)^2}{2d}$$

With the given data values, the force is

$$F = \frac{C(\Delta V)^2}{2d} = \frac{\left(20\times10^{-6}\text{ F}\right)\left(100\text{ V}\right)^2}{2\left(2.0\times10^{-3}\text{ m}\right)} = \boxed{50\text{ N}}$$

16.66 The electric field between the plates is directed downward with magnitude

$$\left|E_y\right| = \frac{\Delta V}{d} = \frac{100\text{ V}}{2.00\times10^{-3}\text{ m}} = 5.00\times10^4\text{ N/m}$$

Since the gravitational force experienced by the electron is negligible in comparison to the electrical force acting on it, the vertical acceleration is

$$a_y = \frac{F_y}{m_e} = \frac{qE_y}{m_e} = \frac{\left(-1.60\times10^{-19}\text{ C}\right)\left(-5.00\times10^4\text{ N/m}\right)}{9.11\times10^{-31}\text{ kg}} = +8.78\times10^{15}\text{ m/s}^2$$

(a) At the closest approach to the bottom plate, $v_y = 0$. Thus, the vertical displacement from point O is found from $v_y^2 = v_{0y}^2 + 2a_y(\Delta y)$ as

$$\Delta y = \frac{0-\left(v_0\sin\theta_0\right)^2}{2a_y} = \frac{-\left[-\left(5.6\times10^6\text{ m/s}\right)\sin 45°\right]^2}{2\left(8.78\times10^{15}\text{ m/s}^2\right)} = -0.89\text{ mm}$$

The minimum distance above the bottom plate is then

$$d = \frac{D}{2} + \Delta y = 1.00\text{ mm} - 0.89\text{ mm} = \boxed{0.11\text{ mm}}$$

(b) The time for the electron to go from point O to the upper plate is found from $\Delta y = v_{0y}t + \frac{1}{2}a_y t^2$ as

$$+1.00\times10^{-3}\text{ m} = \left[-\left(5.6\times10^{6}\ \frac{\text{m}}{\text{s}}\right)\sin 45°\right]t + \frac{1}{2}\left(8.78\times10^{15}\ \frac{\text{m}}{\text{s}^2}\right)t^2$$

Solving for *t* gives a positive solution of $t = 1.11\times10^{-9}$ s. The horizontal displacement from point O at this time is

$$\Delta x = v_{0x}t = \left[\left(5.6\times10^{6}\text{ m/s}\right)\cos 45°\right]\left(1.11\times10^{-9}\text{ s}\right) = \boxed{4.4\text{ mm}}$$